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Decentralized delayed-feedback control of an optimal velocity traffic model

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Abstract. The jam phenomenon in traffic flow wastes not only considerable traffic-transportation time but also great amounts of fuel due to many accelerate-decelerate actions. From traffic-economic and traffic-pollution viewpoints, the suppression of traffic jam is an important issue we have to solve. The present paper shows that H_∞-norm, which has been used in the field of control theory, can reveal the traffic jam phenomenon in an optimal velocity traffic model under an open boundary condition. Furthermore, we suppress the traffic jam in the model by the decentralized delayed-feedback control method. Some numerical simulations are shown to verify our theoretical results.

PACS. 45.70.Vn Granular models of complex systems; traffic flow – 05.45.Gg Control of chaos; applications of chaos – 47.62.+q Flow control

1 Introduction

The traffic flow problems have been widely investigated in the field of physics [1–15]. Several traffic flow models have been proposed: coupled differential equation models [1–8], coupled map models [9–11], and cellular automata [12–15]. Bando *et al.* proposed a car-following traffic model whose individual vehicle is described by a simple nonlinear equation [1]. The vehicle equation is governed by the optimal velocity (OV) function which depends only on the headway distance: an individual driver controls his vehicle velocity on the basis of the OV function. This model is called optimal velocity (OV) traffic model. The paper [1] investigated a traffic jam phenomenon under periodic boundary conditions, and derived a simple stability condition. Komatsu and Sasa examined the traffic jam on the OV model in detail [2]. The OV traffic model was modified as follows: taking into account the delay effect [3–5], modification to be simple and solvable [6], generalization of the model [7], investigation of the open boundary condition [8], and proposal of a discrete-time version model [9–11].

Controlling chaos has gathered much attention of many researchers who are interested in nonlinear dynamics and its applications [16,17]. A delayed-feedback control (DFC) method was proposed as a convenient tool for controlling real chaotic systems [18]. The DFC method does not require a reference signal which is a desired unstable periodic orbit. This feature is a great advantage for experimental situations. The DFC method was successfully used to many physical systems [17]. Several researchers investigated the stability of the DFC system [19–22], and discussed a discrete-time version of the method [23–28]. Most of these studies focused on the stabilization of temporal chaos in low-dimensional systems; on the other hand, investigations of spatiotemporal chaotic behavior and its control have attracted much interest recently [29]. The DFC method was modified to stabilize the spatiotemporal chaotic systems [30–32]. The modified method is the decentralized delayed-feedback control (DDFC) method. Very recently, Konishi, Kokame, and Hirata proposed a dynamic version of the DDFC method, and applied it to suppress the traffic jam in a discrete-time piece-wise linear OV model [33].

Although the mechanism and features of the traffic jam phenomenon in the OV traffic model have been revealed by several theoretical and numerical investigations, to our knowledge, we cannot find the studies on suppression of the phenomenon except the paper [33]. The jam phenomenon wastes not only considerable traffic-transportation time but also great amounts of fuel due to many accelerate-decelerate actions; hence, from traffic-economic and traffic-pollution viewpoints, the suppression of traffic jam should be regarded as an important issue we have to investigate. The present paper shows that H_∞-norm, which has been used in the field of control theory, is a useful tool to analyze the traffic jam phenomenon in the OV traffic models under an open boundary condition. Furthermore, we suppress the traffic jam in the OV traffic model by the continuous-time DDFC method.

This paper is organized as follows. Section 2 explains the OV traffic model, and analyzes its stability.

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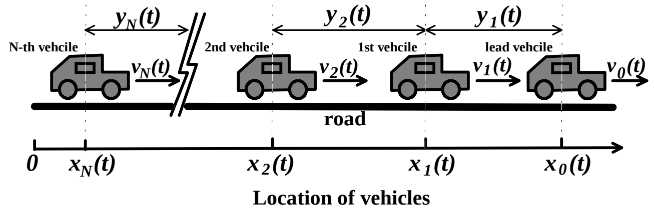


Fig. 1. Illustration of the OV traffic model under an open boundary.

In Section 3, the continuous-time DDFC method is introduced for the suppression of traffic jam, and we give a simple numerical procedure for design of the control system. Section 4 shows numerical simulations to verify the theoretical results. Finally, conclusions are presented in Section 5.

2 Optimal velocity traffic model

2.1 Description of model

Let us consider the optimal velocity (OV) traffic model under an open boundary condition (see Fig. 1). The lead vehicle is described as

$$x_0(t) = v_0 t + x_0(0),$$

where $x_0(t) > 0$ is the position of the leading vehicle, $v_0 > 0$ is its velocity, and $x_0(0) > 0$ is the initial position. We assume that the lead vehicle is not influenced by others. The following vehicles are given as

$$\begin{cases} \dfrac{\mathrm{d}^2 x_i(t)}{\mathrm{d}t^2} = a\left\{F(y_i(t)) - \dfrac{\mathrm{d}x_i(t)}{\mathrm{d}t}\right\}, \\ y_i(t) \quad = x_{i-1}(t) - x_i(t), \end{cases} \quad (i = 1 \sim N), \tag{1}$$

where $x_i(t) > 0$ is the position of the ith vehicle, $y_i(t) > 0$ is the headway distance between the $(i-1)$th and ith vehicles, $a > 0$ is the sensitivity of driver, and N is the number of the following vehicles. $F(y_i(t))$ is the optimal velocity (OV) function which depends only on the headway distance $y_i(t)$.

For the OV models in open boundary conditions, most researchers investigated the vehicle behavior on a road with a finite length: the vehicles enter at the lower boundary of the road and leave at the upper boundary [8,11]. It is not easy to describe the whole system by a simple mathematical form. Hence, in this paper, we focus on the behavior of a vehicle group which consists of vehicles running on a single road without overtaking. System (1) describes such a vehicle group. This is because the group behavior can be treated as the behavior of a group in a long distance traffic road with the open boundary condition. Furthermore, our vehicle group can be described by a simple dynamical equation. From the above reasons, we believe that it is important to reveal the traffic jam mechanism in our traffic system (1).

Letting the velocity of the ith vehicle, $\mathrm{d}x_i(t)/\mathrm{d}t$, be denoted by $v_i(t)$, vehicle dynamics (1) takes the following form:

$$\begin{cases} \dfrac{\mathrm{d}v_i(t)}{\mathrm{d}t} = a\left\{F(y_i(t)) - v_i(t)\right\}, \\ \dfrac{\mathrm{d}y_i(t)}{\mathrm{d}t} = v_{i-1}(t) - v_i(t), \end{cases} \quad (i = 1 \sim N). \tag{2}$$

We shall consider vehicle dynamics (2) instead of (1) throughout this paper. Assume that the lead vehicle runs constantly with speed v_0, then dynamics (2) has the following steady state:

$$\left[v_i^*(t) \;\; y_i^*(t)\right]^{\mathrm{T}} = \left[v_0 \;\; F^{-1}(v_0)\right]^{\mathrm{T}}. \tag{3}$$

This state implies that all the vehicles run orderly with velocity v_0 and headway distance $F^{-1}(v_0)$. Since dynamics (2) has the one-way connection, stability of whole system can be reduced to stability of steady state (3) and transfer function of (2).

2.2 Stability analysis

The linearized system of vehicle system (2) around steady state (3) is

$$\begin{cases} \dfrac{\mathrm{d}\overline{v}_i(t)}{\mathrm{d}t} = a\left\{\Lambda\overline{y}_i(t) - \overline{v}_i(t)\right\}, \\ \dfrac{\mathrm{d}\overline{y}_i(t)}{\mathrm{d}t} = \overline{v}_{i-1}(t) - \overline{v}_i(t), \end{cases} \quad (i = 1 \sim N), \tag{4}$$

where

$$\overline{v}_i(t) := v_i(t) - v_0, \quad \overline{y}_i(t) := y_i(t) - F^{-1}(v_0).$$

Λ is the slope of the OV function at $y_i(t) = F^{-1}(v_0)$:

$$\Lambda := \left.\frac{\partial F(y)}{\partial y}\right|_{y=F^{-1}(v_0)}.$$

From a control theory viewpoint, dynamics (4) can be written as an linear time-invariant system, that is

$$\begin{cases} \begin{bmatrix} \mathrm{d}\overline{v}_i(t)/\mathrm{d}t \\ \mathrm{d}\overline{y}_i(t)/\mathrm{d}t \end{bmatrix} = \begin{bmatrix} -a & a\Lambda \\ -1 & 0 \end{bmatrix} \begin{bmatrix} \overline{v}_i(t) \\ \overline{y}_i(t) \end{bmatrix} + \begin{bmatrix} 0 \\ 1 \end{bmatrix} \overline{v}_{i-1}(t), \\ \overline{v}_i(t) = \begin{bmatrix} 1 & 0 \end{bmatrix} \begin{bmatrix} \overline{v}_i(t) \\ \overline{y}_i(t) \end{bmatrix}. \end{cases} \tag{5}$$

The relation between the $(i-1)$th vehicle velocity disturbance and the ith vehicle velocity disturbance is described by

$$V_i(s) = G(s)V_{i-1}(s), \tag{6}$$

where

$$V_i(s) := \mathcal{L}[\overline{v}_i(t)], \;\; V_{i-1}(s) := \mathcal{L}[\overline{v}_{i-1}(t)].$$

$\mathcal{L}$ denotes the Laplace transformation. The transfer function $G(s)$ is

$$\begin{aligned} G(s) &= \begin{bmatrix} 1 \; 0 \end{bmatrix} \begin{bmatrix} s+a & -a\Lambda \\ 1 & s \end{bmatrix}^{-1} \begin{bmatrix} 0 \\ 1 \end{bmatrix} \\ &= \frac{a\Lambda}{d(s)}, \end{aligned}$$

where the characteristic polynomial $d(s)$ is

$$d(s) := s^2 + as + a\Lambda.$$

In order to simplify our discussion, we give a definition of traffic jam for the OV traffic model.

Definition 1. *Assume that characteristic polynomial $d(s)$ is stable. If H_∞-norm of $G(s)$ is greater than 1, that is*

$$\|G(s)\|_\infty := \sup_{\omega \in [0,+\infty)} |G(j\omega)| > 1,$$

then traffic jam occurs in the OV traffic model.

Now we shall explain Definition 1 in detail. Suppose that the $1, 2, \ldots, (i-1)$th vehicles constantly run with speed v_0. The stability of the polynomial $d(s)$ is the necessary and sufficient condition for steady state (3) to be stable. It is obvious that the ith vehicle can follow the preceding vehicles with velocity v_0 only when steady state (3) is stable. However, this condition does not guarantee that the ith vehicle runs constantly with velocity v_0 when the preceding vehicles velocities fluctuate. Let us assume that the $(i-1)$th vehicle velocity is disturbed as $v_{i-1}(t) = v_0 + \delta_{i-1} \sin(\omega t)$, where δ_{i-1} is a small positive amplitude. Then the ith vehicle velocity disturbance is given as $v_i(t) = v_0 + \delta_i \sin(\omega t + \theta_{i(i-1)})$. The amplitude ratio of δ_i to δ_{i-1} can be described as

$$\frac{\delta_i}{\delta_{i-1}} = |G(j\omega)|\,.$$

For instance, consider the pth and qth vehicles $(q > p)$. The amplitude ratio of δ_q to δ_p is

$$\frac{\delta_q}{\delta_p} = |G(j\omega)|^{q-p} \quad (q > p). \tag{7}$$

Let us consider the following three cases: (i) $|G(j\omega)| < 1$, (ii) $|G(j\omega)| = 1$, (iii) $|G(j\omega)| > 1$. For case (i), the amplitude ratio decreases about exponentially with $q-p$ (see Eq. (7)). Therefore, if the sinusoidal disturbance with angular frequency ω is added to an upper vehicle, this disturbance has little effect on lower vehicles. For case (iii), the amplitude ratio increases about exponentially with $q-p$. If a tiny sinusoidal disturbance with angular frequency ω is added to an upper vehicle, the lower vehicles are significantly influenced by this disturbance. In this case, we can observe the traffic jam phenomenon in the lower vehicle group. For case (ii), this disturbance at an upper vehicle is constantly propagated for all the vehicles. For real traffic systems, we have to consider the external disturbance for all $\omega \in [0, +\infty)$. As a result, we notice that if

$$\|G(s)\|_\infty > 1, \tag{8}$$

then traffic jam occurs in the lower vehicle group. On the contrary, if $d(s)$ is a stable polynomial and condition (8) is not satisfied, we cannot observe the traffic jam phenomenon. Now we show a simple condition under which traffic jam never occurs.

Lemma 1. *If the following condition is satisfied:*

$$2\Lambda < a, \tag{9}$$

then the traffic jam never occurs in the OV traffic model.

Proof. From Definition 1, we know that the traffic jam never occurs when the following two conditions are satisfied: (1) $\|G(s)\|_\infty$ is 1 or less; (2) $d(s)$ is a stable polynomial. The absolute value of the transfer function $G(j\omega)$ is

$$\begin{aligned} |G(j\omega)| &= \sqrt{G(j\omega)G(-j\omega)} \\ &= \sqrt{a^2\Lambda^2/g(\omega)}, \end{aligned}$$

where $g(\omega) = (a\Lambda - \omega^2)^2 + \omega^2 a^2$. Condition (1) holds if the following three conditions are satisfied: (1-1) $|G(j0)| = 1$; (1-2) $|G(j\infty)| = 0$; (1-3) $\partial g(\omega)/\partial\omega \neq 0$ for all $\omega \in (0, \infty)$. It is obvious that conditions (1-1) and (1-2) are satisfied for any parameters. Condition (1-3) can be described as $\partial g(\omega)/\partial\omega = 2\omega(2\omega^2 + a^2 - 2a\Lambda) \neq 0$ for all $\omega \in (0, \infty)$. This is satisfied if and only if condition (9) holds. Furthermore, condition (2) is always satisfied when condition (9) holds. □

It should be noted that condition (9) is the same as the stability of the OV traffic model under periodic boundary conditions derived in [1].

3 Suppression of traffic jam

Let us add a control signal term, $u_i(t)$, to vehicle dynamics (1):

$$\begin{cases} \dfrac{\mathrm{d}^2 x_i(t)}{\mathrm{d}t^2} = a\left\{F(y_i(t)) - \dfrac{\mathrm{d}x_i(t)}{\mathrm{d}t}\right\} + u_i(t), \\ y_i(t) = x_{i-1}(t) - x_i(t), \end{cases} \quad (i = 1 \sim N),$$

where the control signal $u_i(t)$ is

$$u_i(t) = k\left(y_i(t) - y_i(t-\tau)\right). \tag{10}$$

k, τ are the feedback gain and delay time respectively. The control signal $u_i(t)$ is proportional to the difference between the present headway distance $y(t)$ and the past one

$y(t-\tau)$. System (5) can be rewritten as

$$\begin{cases} \dfrac{\mathrm{d}v_i(t)}{\mathrm{d}t} = a\left\{F(y_i(t)) - v_i(t)\right\} + u_i(t), \\ \dfrac{\mathrm{d}y_i(t)}{\mathrm{d}t} = v_{i-1}(t) - v_i(t), \end{cases} \quad (i = 1 \sim N). \tag{11}$$

Control system (11) is a continuous-time version of the DDFC proposed in [30]. We note that the control signal $u_i(t)$ vanishes when the ith vehicle is running with a constant velocity. Around the steady state (3), control system (11) and controller (10) are written as a linear time-invariant system, that is

$$\begin{cases} \begin{bmatrix} \mathrm{d}\overline{v}_i(t)/\mathrm{d}t \\ \mathrm{d}\overline{y}_i(t)/\mathrm{d}t \end{bmatrix} = \begin{bmatrix} -a & a\Lambda \\ -1 & 0 \end{bmatrix} \begin{bmatrix} \overline{v}_i(t) \\ \overline{y}_i(t) \end{bmatrix} + \begin{bmatrix} 0 \\ 1 \end{bmatrix} \overline{v}_{i-1}(t) + \begin{bmatrix} 1 \\ 0 \end{bmatrix} u_i(t), \\ \overline{v}_i(t) = \begin{bmatrix} 1 & 0 \end{bmatrix} \begin{bmatrix} \overline{v}_i(t) \\ \overline{y}_i(t) \end{bmatrix}, \\ \overline{y}_i(t) = \begin{bmatrix} 0 & 1 \end{bmatrix} \begin{bmatrix} \overline{v}_i(t) \\ \overline{y}_i(t) \end{bmatrix}, \end{cases}$$

where

$$\begin{aligned} u_i(t) &= k\left(y_i(t) - y_i(t-\tau)\right) \\ &= k\left(\overline{y}_i(t) - \overline{y}_i(t-\tau)\right). \end{aligned}$$

From frequency domain viewpoint, this linearized system can be written as

$$\begin{aligned} V_i(s) &= G_{11}(s)V_{i-1}(s) + G_{12}(s)U_i(s), \\ Y_i(s) &= G_{21}(s)V_{i-1}(s) + G_{22}(s)U_i(s), \\ U_i(s) &= H(s)Y_i(s), \end{aligned}$$

where

$$\begin{aligned} &Y_i(s) := \mathcal{L}[\overline{y}_i(t)], \quad U_i(s) := \mathcal{L}[u_i(t)], \\ &G_{11}(s) := \frac{a\Lambda}{d(s)}, \quad G_{12}(s) := \frac{s}{d(s)}, \\ &G_{21}(s) := \frac{s+a}{d(s)}, \quad G_{22}(s) := -\frac{1}{d(s)}, \\ &H(s) := k(1 - \mathrm{e}^{-s\tau}). \end{aligned}$$

Figure 2 illustrates the block diagram of this control system. The relation between $V_{i-1}(s)$ and $V_i(s)$ is described as

$$V_i(s) = \overline{G}(s)V_{i-1}(s). \tag{12}$$

The transfer function $\overline{G}(s)$ is given by

$$\begin{aligned} \overline{G}(s) &= G_{11}(s) + G_{12}(s)H(s)\left(1 - G_{22}(s)H(s)\right)^{-1} G_{12}(s) \\ &= \frac{a\Lambda + k(1-\mathrm{e}^{-s\tau})}{d(s) + k(1-\mathrm{e}^{-s\tau})}. \end{aligned}$$

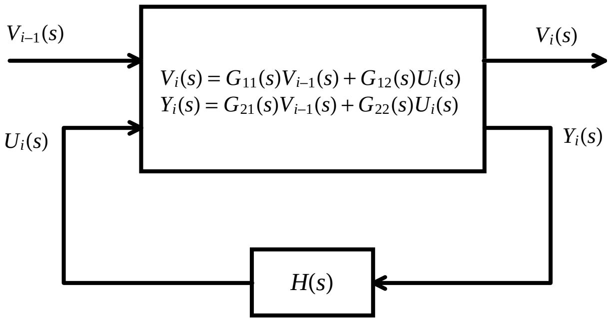


Fig. 2. Block diagram of the control system for the ith vehicle.

We note that equation (12) corresponds to equation (6); therefore, from Definition 1, the traffic jam never occurs in the OV traffic model if the characteristic polynomial of $\overline{G}(s)$, that is

$$\overline{d}(s) := d(s) + k(1 - \mathrm{e}^{-s\tau}),$$

is stable and the $\|\overline{G}(s)\|_\infty$ is 1 or less.

The main purpose of this section is to provide a way how to design the feedback gain k and the delay time τ such that $\overline{d}(s)$ is stable and $\|\overline{G}(s)\|_\infty$ is 1 or less.

First of all, we shall give a condition for $\overline{d}(s)$ to be stable. The well-known small gain theorem guarantees that $\overline{d}(s)$ is stable if

$$\|G_{22}(s)\|_\infty \|H(s)\|_\infty < 1. \tag{13}$$

The absolute value of $G_{22}(s)$ is

$$|G_{22}(j\omega)| = \frac{1}{\sqrt{h(\omega)}},$$

where

$$h(\omega) := w^4 + a(a - 2\Lambda)w^2 + a^2\Lambda^2.$$

When the traffic jam occurs in the OV model without control (*i.e.*, $2\Lambda < a$), we have

$$\inf_{\omega\in[0,+\infty)} h(\omega) = \frac{a^2(3a^2 - 12a\Lambda + 16\Lambda^2)}{4}.$$

Hence, H_∞-norm of $G_{22}(s)$ is

$$\|G_{22}(s)\|_\infty = \frac{2}{a\sqrt{3a^2 - 12a\Lambda + 16\Lambda^2}}.$$

On the other hand, it is easy to derive H_∞-norm of $H(s)$:

$$\|H(s)\|_\infty = \|k(1 - \mathrm{e}^{-s\tau})\|_\infty = 2|k|.$$

From small gain theorem (13), we notice that $\overline{d}(s)$ is stable if the feedback gain k is chosen as

$$|k| < \frac{a\sqrt{3a^2 - 12a\Lambda + 16\Lambda^2}}{4}. \tag{14}$$

It should be noted that this condition does not depend on the delay time τ.

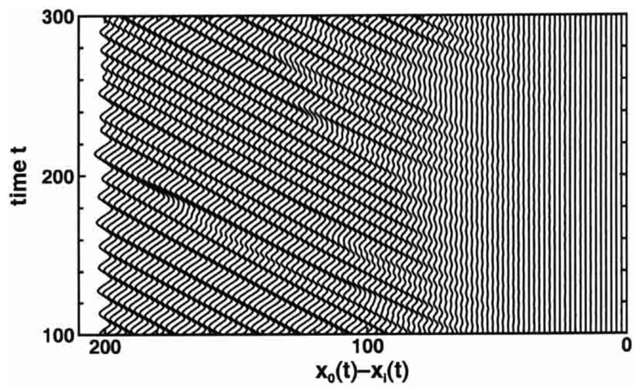
Secondly, we have to design k and τ such that $\|\overline{G}(s)\|_\infty$ is 1 or less. The absolute value of $\overline{G}(s)$ can be described as

$$\left|\overline{G}(j\omega)\right| = \frac{\{a\Lambda + k(1-\cos\omega\tau)\}^2 + k^2 \sin\omega\tau^2}{\{a\Lambda - \omega^2 + k(1-\cos\omega\tau)\}^2 + (\omega a + k\sin\omega\tau)^2}\cdot \quad (15)$$

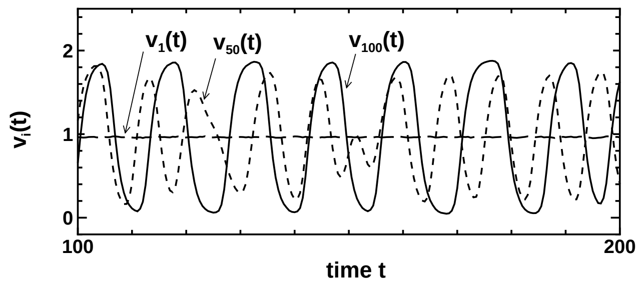
Since it is difficult to derive H_∞-norm of $\overline{G}(s)$ analytically, we have to determine the gain k and the delay τ such that $\left|\overline{G}(j\omega)\right|$ is 1 or less for all $\omega \in [0, +\infty)$ on numerical simulations. A design procedure is summarized as follows.

Theorem 1. *Assume that the traffic jam occurs in the OV model without control (i.e., $2\Lambda > a$). If the feedback gain k and the delay time τ satisfy condition (14) and $\left|\overline{G}(j\omega)\right|$ is 1 or less for all $\omega \in [0, +\infty)$ (i.e., $\|\overline{G}(s)\|_\infty \leq 1$), then the traffic jam never occurs in the OV model with control.*

Here we have a problem of checking $\|\overline{G}(s)\|_\infty \leq 1$ in Theorem 1 numerically. In order to solve this problem, we should achieve the following procedure: (i) draw the gain diagram of $\overline{G}(s)$ by using equation (15); (ii) judge whether the peak gain of the diagram is not more than 1. It is easy to achieve this procedure on computers.



(a)



(b)

Fig. 3. Numerical simulation for the free-running traffic model. (a) Space-time plot. (b) Temporal velocity behavior of three vehicles.

4 Numerical simulations

Let us consider a case where 100 vehicles are running on a single road without overtaking under an open boundary. The OV function is set as [1]

$$F(y) = \tanh(y - x_c) + \tanh(x_c),$$

where x_c is a desired headway distance. The parameters are set as $a = 1.0, x_c = 2.0, v_0 = 0.964$. The OV function and these parameters in our simulations are the same as the paper [1]. For all simulation results, we use Runge-Kutta algorithm for numerical integration with time step $\Delta t = 0.01$. The uniform random noise with maximum amplitude 10^{-3} is added to the first equation of (2) for all the vehicles.

First of all, we consider the OV traffic model without control (*i.e.*, $k = 0$ or $\tau = 0$) on computer simulations. We show the space-time plot of the running traffic model after $t = 100$ in Figure 3a. The horizontal axis represents a distance between the lead vehicle and a following vehicle. The vertical axis indicates the time development. It can be seen that the upper vehicle group (see the right part of Fig. 3a) run constantly with the lead vehicle velocity v_0; however, we observe the oscillating headway distances in the lower vehicle group (see the left part of Fig. 3a). Figure 3b shows the velocity behavior of the 1st, 50th, and 100th vehicles. The 1st vehicle runs constantly with velocity v_0. The 50th vehicle velocity oscillates with accelerate-decelerate actions. The 100th vehicle velocity also oscillates, and the amplitude of this oscillation is larger than that of 50th vehicle. These numerical results mean that the traffic jam occurs in the OV traffic model.

Secondly, in order to suppress the traffic jam, we have to determine the feedback gain k and the delay time τ. From Theorem 1, we should choose the feedback gain k such that it satisfies condition (14). Substituting the parameter values into condition (14), we obtain $|k| < 0.661437$. Let us fix the gain at $k = 0.5$. Figure 4a shows the absolute values of the transfer function $\left|\overline{G}(j\omega)\right|$, which is described by equation (15), for $\tau = 0$, 1, 2. The absolute value for $\tau = 0$, which is the same as that with no control, has a peak greater than 1. The value for $\tau = 2$ has also a peak greater than 1. The value for $\tau = 1$ is 1 or less for all $\omega \in [0, +\infty)$; hence, the delay time $\tau = 1$ satisfies the condition of Theorem 1. Furthermore, we estimate the absolute values for $k = -0.5$, 0.0, +0.5 when the delay time is fixed as $\tau = 1.0$ (see Fig. 4b). As you see, we find that the value for $k = 0.5$ is 1 or less for all $\omega \in [0, +\infty)$. These numerical calculations guarantee that the traffic jam never occurs in the OV traffic model with the gain $k = 0.5$ and the delay time $\tau = 1$.

Thirdly, we simulate the controlled traffic model. Figure 5a shows the space-time plot of the controlled traffic model after $t = 100$. There is no traffic jam in the OV model. Figure 5b indicates the velocities of the 1st, 50th, and 100th vehicles. It can be seen that all the vehicles

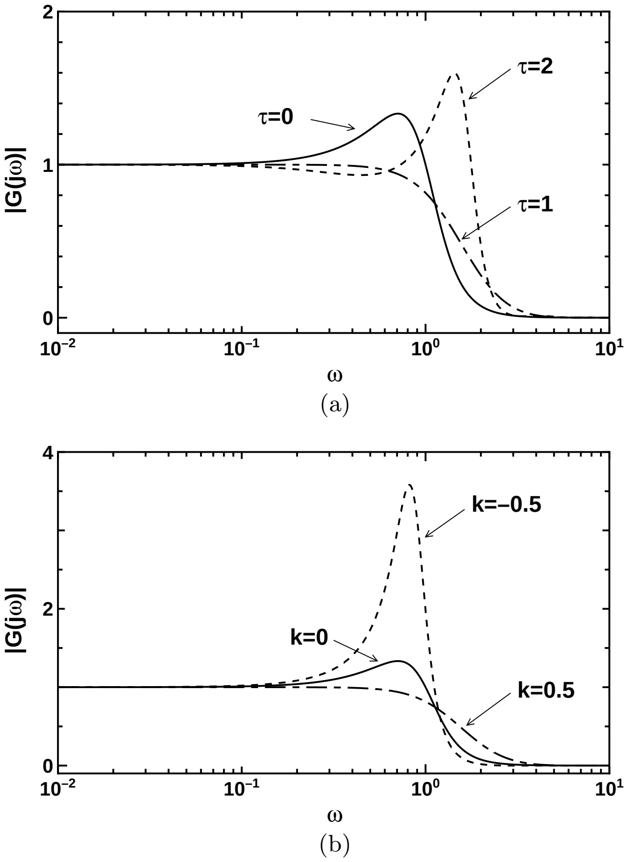


Fig. 4. Gain diagram of $\overline{G}(j\omega)$. (a) $k = 0.5$. (b) $\tau = 1$.

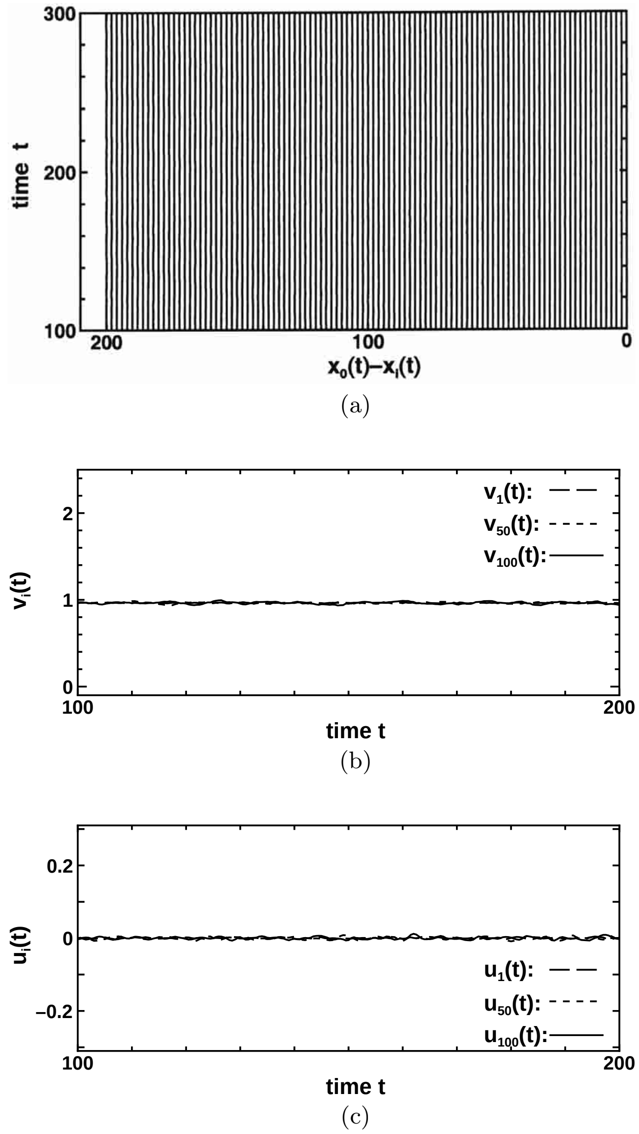


Fig. 5. Behavior of controlled traffic model. (a) Space-time plot. (b) Temporal velocity behavior of three vehicles. (c) Control signals of three vehicles.

run constantly with speed v_0. The control signals of the 1st, 50th, and 100th vehicles are shown in Figure 5c. We can confirm that all the control signals are almost zero. This numerical simulation substantiates that our theoretical results are useful to suppress the traffic jam in the OV model.

Finally, we consider the step external disturbance: the lead vehicle slows down twice, that is

$$v_0(t) = \begin{cases} v_0/2 & \text{if} \quad 110 \le t \le 115, \\ v_0/2 & \text{if} \quad 130 \le t \le 135, \\ v_0 & \text{if} \quad \text{otherwise.} \end{cases}$$

The space-time plot of the no-control traffic model after $t = 100$ is shown in Figure 6a. Figure 6b shows the velocity behavior of three vehicles. We find the traffic jam in the OV traffic model. Figure 7a indicates the space-time plot of the controlled traffic model. We see that there is no traffic jam in the OV model. Figures 7b and 7c show the temporal velocity behavior and the control signals for the three vehicles. The minimum velocity and the amplitude of the control signal decrease with increasing vehicle number i. From this numerical simulation, we can say that the controlled traffic model is robust for external disturbances.

5 Conclusions

This paper showed that the continuous-time DDFC method suppresses the traffic jam phenomenon in the OV traffic model. We derived a procedure to design the feedback gain k such that the controlled traffic system to be stable. Furthermore, we provide a numerical way to check whether the feedback gain k and the delay time τ can hold the H_∞-norm of each vehicle transfer function to 1 or less. Our control scheme has the following four advantages: (i) The controller of each vehicle does not require other vehicle information (*e.g.*, other vehicle-velocity, -position, -parameters, and so on); (ii) Each controller does not need a desired velocity (*i.e.*, the lead vehicle velocity v_0); (iii) Our control scheme is useful for any size traffic model;

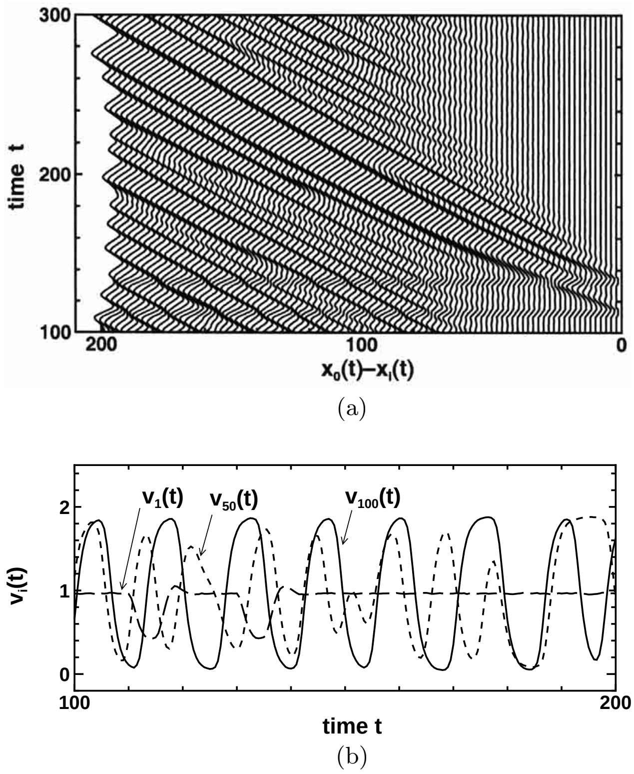


Fig. 6. Behavior of no-control traffic model. The lead vehicle slows down twice. (a) Space-time plot. (b) Temporal velocity behavior of three vehicles.

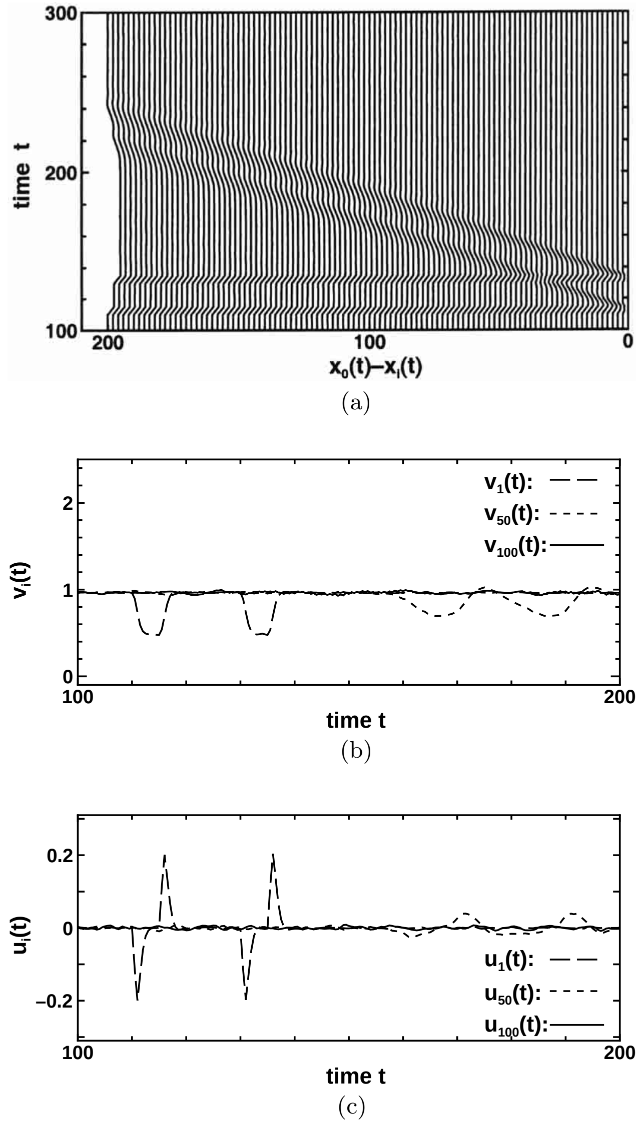


Fig. 7. Behavior of controlled traffic model. The lead vehicle slows down twice. (a) Space-time plot. (b) Temporal velocity behavior of three vehicles. (c) Control signals of three vehicles.

(iv) There is no need to change the vehicle parameters. These advantages are practical for real traffic flows. In addition, we showed that the numerical simulations agree well with our theoretical results.

It would be important to consider how to use our results for practical situations. Unfortunately, we can not provide a concrete scheme to realize the control signal $u_i(t)$ right now. We plan in the future to study the realization of $u_i(t)$ for practical situations.

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